- **Derivatives of trigonometric functions** Derivatieves of Sine and Cosine.

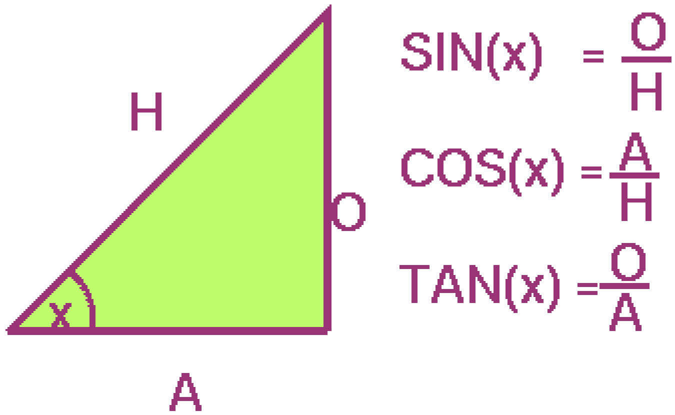
Main Part of class

We will find the derivatives of sine and cosine functions, and see them in some examples.

Definition of Trigonometric functions

Trigonometric functions are used for measuring angles. There are three main trigonometric functions, sine, cosine, and tangent, called sin, cos and tan for short; if the angle is x, write sin(x), cos(x), tan(x).

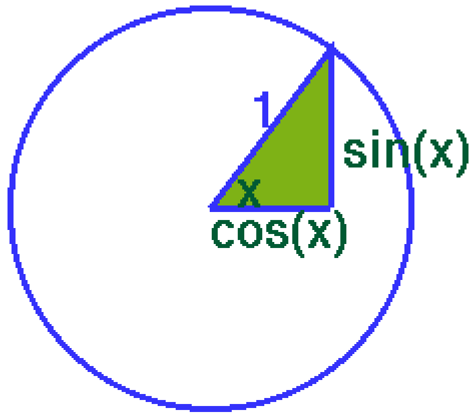
The following diagram describes how these are defined:



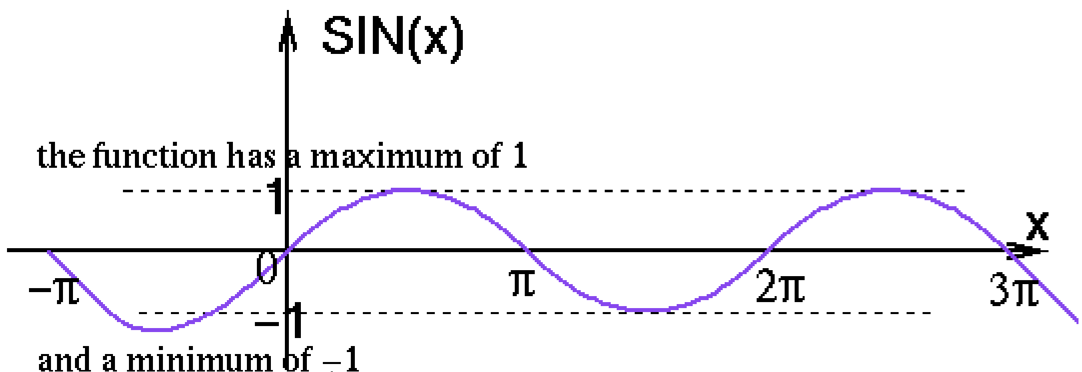
The inverse trigonometric functions tell you what angle has a certain sine, what value has a certain cosine, eg, what angle has sine(x) = 1/2 ? The answer is 60 degrees, so we say $\sin^{-1}(1/2) = 60$

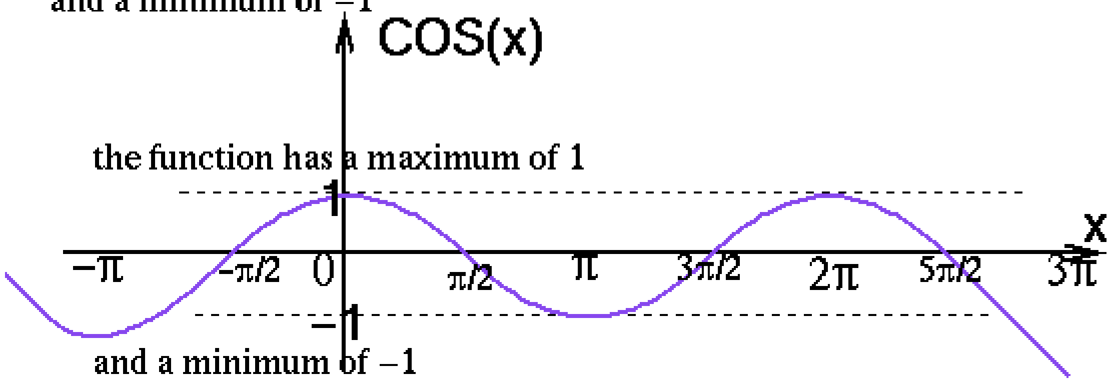
The graphs of trigonometric functions.

To picture the graph of how sine(x) and cosine(x) change as the angle x changes, put the triangle in a circle:



The graphs of cosine and sine are periodic, so they are very useful for describing all kinds of periodic behaviour:





Derivatives of trigonometric functions

The derivative of trigonometric functions tells us the rate of change of the periodic behaviour that's being described.

You need to learn these rules for finding the derivatives:

d/dx(sin(x)) = cos(x)

$$d/dx(\cos(x)) = -\sin(x)$$

In the text book proofs of these formulae are given, but we are not going to cover that. There are also derivatives of other trigonometric functions

The important thing to note is that the derivatives of the trigonometric funtions are simply related to each other, and by examining the graphs, we can understand why this is.

Combining trigonometric functions with other functions

We will also cover some examples where trigonometric functions are combined with other functions, and see what these functions look like, and what are their derivatives, eg:

- $x * \sin(x)$
- $\sin(x + 1)$
- $\sin(x^2)$

You can find the derivatives of these using product rule, chain rule, etc.

Rules for dealing with trig functions

We can do all the same kinds of problems for trigonometric functions as we did for polynomial functions, eg, find equations for tangents, or maximum and minimum values. But trigonometric functions are a little more difficult than polynomial functions, and there are a few problems that come up. The following are the main things that we'll use to solve these problems.

- Double angle formulae
- How to find inverse values of trig functions

Angle formulae

Here are formulae you may need if you have an equation involving something like sin(x) and sin(2x) at the same time; this is a way to compare sine and cosine of different angles:

$$SIN(A + B) = SIN(A)*COS(B) + COS(A)*SIN(B)$$
$$COS(A + B) = COS(A)*COS(B) - SIN(A)*SIN(B)$$

Another even more important formula to remember, which comes straight from Pythagoras' theorem is

$$SIN^2(A) + COS^2(A) = 1$$

I will be putting up examples to show how these rules can be used in calculus soon.

Inverse of trigonometric functions

Inverses are about solving equations.

For example, if you want to solve

$$x^2 = 4,$$

you take the sqaure root, and get

$$x = 4^{1/2} = 2$$

Generally, if

$$x^2 = a,$$

then

$$x = a^{1/2}$$

So, the square root function gives you the solution to the square function. We describe this situation by saying that

the square root function is the **inverse** of the square function.

For any function f(x), the inverse is called $f^{-1}(x)$, this means that

If f(x) = A, then the solution is $x = f^{-1}(A)$

For example, if the function is

$$f(x) = x^2 + 4x + 1 \text{ ?}$$

Suppose we want to find when this function has a particular value, say 7. We want to solve

$$f(x) = x^2 + 4x + 1 = 7$$

We rewrite this as

$$f(x) = x^2 + 4x + 1\text{-}7 =$$

And then we can solve it using the quadratic formula:

$$x = (-4 + (16 - 4(1-7))^{1/2})/2$$

We can do the same thing for any number A,

If $x^2 + 4x + 1 = A$ Then one possible solution is $x = (-4 + (16 - 4(1-A))^{1/2})/2$

In inverse notation, we have

If $f(x) = x^2 + 4x + 1 = A$ Then $f^{-1}(A) = (-4 + (16 - 4(1-A))^{1/2})/2$

What about the other solutions?

You might notice that in all these examples I am cheating, since I'm only giving you one possible solution; eg, for $x^2=4$, x = 2 is a solution, but x = -2 is also a solution. **But** in this case, if you know one solution, it's easy to get the other solution - you just mulitply by -1.

Generally, if we have one solution, it would be nice if we could find other solutions from that one solution in an easy way, just like this multiplication by -1 is pretty easy, once you've got one solution.

How to find infinitely many solutions to sin(X) = A

You can find this from looking at the graph, and I'm going to write out a detailed explaination here later.

The idea is:

If sin(X) = A, then we also have sin(2pi + X) = A, sin(4pi + X) = A, sin(6pi + X) = A, sin(8pi + X) = A, sin(pi - X) = A, sin(3pi - X) = A, sin(5pi - X) = A, sin(7pi - X) = A,
If cos(X) = A, then we also have sin(2pi + X) = A, sin(4pi + X) = A, sin(6pi + X) = A, sin(8pi + X) = A, sin(2pi - X) = A, sin(4pi - X) = A, sin(6pi - X) = A, sin(8pi - X) = A,

This can be understood by examining the graphs above.
