

Test 4
MTH 251

1. Evaluate each integral or show that it diverges.

a. $\int_0^3 \frac{1}{(x-3)^2} dx$

b. $\int_0^{\infty} e^{-(2x+1)} dx$

c. $\int_{-2}^0 \frac{dx}{\sqrt{x+2}}$

d. $\int_2^{\infty} \frac{1}{x(\ln x)^{2/3}} dx$

2. Evaluate each limit (write ∞ or $-\infty$ where appropriate.)

a. $\lim_{x \rightarrow 0} \frac{\sin(x) - x}{x^3}$

b. $\lim_{x \rightarrow \infty} x \sin(1/x)$

c. $\lim_{x \rightarrow 0^+} x \ln x$

d. $\lim_{x \rightarrow \infty} (\ln x - x)$

e. $\lim_{x \rightarrow \infty} \frac{\sqrt{x^6 - x^3 + 5}}{2x^3 + 3}$

3. Determine whether the sequence converges or diverges. If it converges, find the limit.

a. $a_n = \sqrt{n+3} - \sqrt{n}$

b. $a_n = (1 + 3n)^{1/n}$

c. $a_n = \frac{\cos^2 n}{n^3}$

4. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

a. $\sum_{n=0}^{\infty} \frac{4^{n+1}}{5^n}$

b. $\sum_{n=1}^{\infty} \frac{4+3^n}{2^n}$

5. The series $\sum_{k=0}^{\infty} \frac{x^k}{3^k}$ converges only for

- a. $-1 < x < 1$
- b. all x
- c. $x = 3$
- d. $-3 < x < 3$
- e. None of these