Quiz 1 October 8, 2004

Probability Theory and Applications

Name:________________________

Present your work **neatly** in the working space provided. For additional space use back of the page. Mark each part and circle your answer whenever possible. Answers should be in fractional or decimal form unless otherwise indicated.

Binomial and Poisson tables are attached.

Problem No.	**Points**	
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

The final exam has been moved to 10 a.m., Monday December 20

—–, Yes, I will take exam 10 a.m., Monday December 20

—-, No, I have a conflict with another exam and will need to take exam at the originally scheduled time (6:30 p.m., Monday December 20).

1. (20 pts) Let X be a discrete random variable with a probability density function of

x	-3	-1	0	1	2	3	5	8
f(x)	.1	.2	.15	.2	a	.15	.05	.05

(a) (5 points) What is the value of a?

(b) (5 points) What is P(X=3.4)?

(C) (10 points) Sketch the c.d.f of f. What is the $P(X \leq 3)$ and what is $P(X \leq 9)$?

2. (20 pts) The School of Science at Fine College consists of four departments: Math, Biology, Physics, and Computer Science. Math and Computer Science each have 20 professors each. Biology and Physics each have 10 professors each. The dean creates a committe with three members by first randomly selecting three departments and then randomly selecting one professor from each of the three departments to be on the committee.

(a) (8 points) How many committees are possible?

(b) (8 points) Are all committees equally likely? Give yes or no answer and support your answer mathematically.

(c)(4 points) Let A be the event that Dr. Bennett from the Math Department serves on the committee. Let B be the event that Dr. Holmes from the Math Department serves on the committee. Are A and B independent? Why or why not?

3. (20 pts) Assume Great University has 6000 students. A random sample of 300 different students living in university housing is selected. Each of the 300 students are asked whether they plan to vote for Ralph Nader or not. 5 students answer yes they will vote for Ralph Nader. The rest say no.

(a) (8 points) Assume the proportion of the students planing to vote for Nader is p. What should the value of p be such that such that the expected number of students who plan to vote for Nader in the poll is 5?

(b) (8 points) Assume the actual value of p is .03. What is the probability that no more than five students will say they plan to vote for Nader in the Poll?

(c) (4 points) Do you think the percentage of students saying yes to this poll is an accurate estimate of the percentage of students who plan to vote for Nader at Great University? Why or why not?

4. (20 pts) An RNA codon consists of three nucleotides. Each nucleotide is represented by one of four letters: A, C, U, or G. Using the "genetic code", the RNA codon is translated into an amino acid. An amino acid may be produced by more than one codon. Say a biologist is interested in four amino acids: Leucine, Tyrosine, Arginine and Serine. Leucine can be translated by 6 codons: UUA, UUG, CUU, CUC, CUA, and CUG. Tyrosine can be translated from two codons UAU and UAC. Arginine can be translated from four codons: CGU, CGC, CGA, CGG. Serine can be translated from two codons: CAU and CAC.

Assume that all possible exist in practice and codons are equally likely.

a) Given that a codon if for one of the four amino acids (Leucine, Tyrosin, Arginine, or Serine) and that it starts with the nucleotide C, what is the probability that it translates for Leucine?

b) Say you looking at a bunch of codons that are assumed to be independent. You are looking for a codon that translates for Leucine, Tyrosine, Arginine, or Serine. As soon as you find one you will stop. You have already looked at 5 codons without finding a Leucine, Tyrosine, Arginine or Serine. What is the probability you will examine at least 10 more codons before you see one that is either Leucine, Tyrosine, Arginine, or Serine?

5. (20 pts) Airlines frequently overbook flights, e.g. they sell more tickets for the flight then there are seats in the plane. Assume that the probability that a passenger actually shows up for a flight that he/she has a ticket for is 0.8. Assume all passengers act independently.

A flight can fit at most 5 passengers. If more than 5 passengers take the plane then they will be overbooked and they will have to compensate the remaining passengers with a free ticket.

(a) (4 point) Say the airline sells 6 tickets, what is the probability that they will be overbooked? (This is the same as saying that all 6 passengers will show up for the flight)

(b) (8 points) The airline decided that they can tolerate no more than a 10 percent chance of overbooking the flight by one or more passengers. How many tickets should they sell?

(c) (8 points) Assume the airline makes \$1000 for every passenger who buys a ticket for the flight, and they lose \$100 if a passenger is bumped. If they sell 7 seats, what is the expected profit for that flight? What is the variance in the profit?